

A remark on Fano 4-folds having (3,1)-type extremal contractions

Toru Tsukioka

February 2, 2008

Abstract

Let X be the blow-up of a smooth projective 4-fold Y along a smooth curve C and let E be the exceptional divisor. Assume that X is a Fano manifold and has an elementary extremal contraction $\varphi : X \rightarrow Z$ of (3,1)-type (i.e. the exceptional locus of φ is a divisor and its image is a curve) such that E is φ -ample. We show that if the exceptional divisor of φ is smooth, then Y is isomorphic to \mathbb{P}^4 and C is an elliptic curve of degree 4 in \mathbb{P}^4 .

1 Introduction

As an application of the extremal contraction theory, S. Mori and S. Mukai classified smooth Fano 3-folds with Picard number greater than or equal to 2 ([MM]). We observe that many of examples in the Mori-Mukai's list are obtained by blowing up other smooth projective 3-folds. In fact, 78 types among 88 types of smooth Fano 3-folds with $\rho \geq 2$ have E_1 -type or E_2 -type extremal contractions. In [BCW] the authors classified smooth Fano varieties (defined over \mathbb{C}) obtained by blowing-up a smooth point, in any dimension. A next step is to consider the following problem:

Problem. Let Y be a smooth projective variety. Let $\pi : X \rightarrow Y$ be the blow-up along a smooth curve C . Classify pairs (Y, C) such that X is Fano.

Remark that for the toric case, the classification is done in any dimension by [S].

By the Cone and Contraction Theorem, we can take an extremal contraction $\varphi : X \rightarrow Z$ to normal projective variety such that the exceptional divisor E of π is φ -ample (see Lemma 1 below). It is easy to show that any fiber of φ is at most of dimension 2. The author studied the case where φ is a del Pezzo surface fibration and gave a complete classification ([T2]). In higher dimensions, it seems difficult to classify the case where φ is birational. However, in dimension 4, there are several results on the birational extremal contractions, which may be applied to solve our problem.

In this paper, we investigate the case where φ is of $(3, 1)$ -type contraction. Recall that in general, an extremal contraction $\varphi : X \rightarrow Z$ is said to be (a, b) -type, if $\dim(\text{Exc}(\varphi)) = a$ and $\dim(\varphi(\text{Exc}(\varphi))) = b$. So, a $(3, 1)$ -type contraction for a 4-fold is a birational contraction which contracts a divisor F to a curve B . The extremal contractions of $(3, 1)$ -type for smooth 4-folds are completely classified by [Tk]. In particular, it is shown that the exceptional divisor F is normal and B is smooth. Moreover, $\varphi|_F : F \rightarrow B$ is either a \mathbb{P}^2 -bundle or a Q_2 -bundle (see [Tk] Main Theorem).¹

In section 2, we first give an example. Let $C \subset \mathbb{P}^4$ be a smooth complete intersection of one hyperplane and two hyperquadrics. Then, we see that $X = \text{Bl}_C(\mathbb{P}^4)$ has a $(3, 1)$ -type extremal contraction to a complete intersection of two hyperquadrics (singular along a line) in \mathbb{P}^6 . The section 3 is devoted to show that this is the only example if we assume that $\text{Exc}(\varphi)$ is smooth. More precisely, we prove the following:

Theorem 1. *Let $\pi : X \rightarrow Y$ be the blow-up of a smooth projective 4-fold Y defined over \mathbb{C} , along a smooth curve C . Assume that X is a Fano manifold and has an elementary extremal contraction $\varphi : X \rightarrow Z$ of $(3, 1)$ -type such that the exceptional divisor E of π is φ -ample. Let F be the exceptional divisor of φ . If F is smooth, then Y is isomorphic to \mathbb{P}^4 and C is a smooth complete intersection of a hyperplane and two hyperquadrics.*

We will use the following lemma, which is essentially the same as in [BCW](Lemme 2.1). For reader's convenience, we include here the statement with its proof.

Lemma 1. *Let X be a Fano manifold and let E be a non-zero effective divisor on X . Then there exists an extremal ray $\mathbb{R}^+[f] \subset \overline{\text{NE}}(X)$ such that $E \cdot f > 0$.*

Proof. Since X is projective, we can take a curve Γ on X such that $E \cdot \Gamma > 0$. By the Cone Theorem, there exist positive real numbers a_i , and extremal rational curves f_i such that $\Gamma \equiv \sum a_i f_i$ (finite sum). Hence

$$0 < E \cdot \Gamma = \sum a_i (E \cdot f_i).$$

This implies that one of extremal rational curves satisfies $E \cdot f_i > 0$. ■

Throughout this paper, we shall assume that the base field is the complex numbers. For a Cartier divisor D and a 1-cycle α on a variety X , we denote the intersection number by $D \cdot \alpha$, but we also write $(D \cdot \alpha)_X$ when we need to clarify the variety in which the intersection number is taken.

¹Remark that our F and B correspond to E and C in [Tk].

2 An example

We give an example of a smooth Fano 4-fold X obtained by blowing up along a curve such that X has another $(3, 1)$ -type extremal contraction.

Example Let $C \subset \mathbb{P}^4$ be a smooth complete intersection of a hyperplane and two hyperquadrics, $\pi : X \rightarrow \mathbb{P}^4$ the blow-up along C , and E the exceptional divisor. Let F be the strict transform of the hyperplane containing C . Remark that $F \simeq \text{Bl}_C(\mathbb{P}^3)$ is a Q_2 -bundle over \mathbb{P}^1 . Let e be a line in a fiber of the \mathbb{P}^2 -bundle $\pi|_E : E \rightarrow C$, and let f be the strict transform of a line in \mathbb{P}^4 intersecting C at two points. Then we have

$$\overline{\text{NE}}(X) = \mathbb{R}^+[e] + \mathbb{R}^+[f].$$

The extremal contraction associated to the ray $\mathbb{R}^+[e]$ is of course the blow-up $\pi : X \rightarrow \mathbb{P}^4$. Let $L := \pi^*\mathcal{O}_{\mathbb{P}^4}(1)$. The linear system $|2L - E|$ is base-point-free and defines the extremal contraction $\varphi : X \rightarrow Z$ of the ray $\mathbb{R}^+[f]$. Indeed, we have $(2L - E) \cdot f = 0$. Note that $B := \varphi(F)$ is isomorphic to \mathbb{P}^1 and $\varphi|_F : F \rightarrow B$ is a Q_2 -bundle. Thus, φ is a $(3, 1)$ -type extremal contraction whose exceptional divisor is F . More precisely, the image Z is a complete intersection of two hyperquadrics in \mathbb{P}^6 , singular along $B \simeq \mathbb{P}^1$. To see this, we calculate $h^0(X, \mathcal{O}_X(2L - E))$ and $(2L - E)^4$.

Consider the exact sequence:

$$0 \rightarrow \mathcal{O}_X(2L - E) \rightarrow \mathcal{O}_X(2L) \rightarrow \mathcal{O}_E(2L) \rightarrow 0.$$

Remark that $A := -K_X + (2L - E) = (5L - 2E) + (2L - E) = 7L - 3E$ is ample by Kleiman's criterion, because $A \cdot e = 3 > 0$ and $A \cdot f = 1 > 0$. Therefore, by the Kodaira vanishing, $H^1(X, \mathcal{O}_X(2L - E)) = 0$. On the other hand, we get $h^0(X, \mathcal{O}_X(2L)) = h^0(\mathbb{P}^4, \mathcal{O}_{\mathbb{P}^4}(2)) = 15$. Since $\mathcal{O}_E(2L) \simeq (\pi|_E)^*\mathcal{O}_C(2)$, we have $h^0(E, \mathcal{O}_E(2L)) = h^0(C, \mathcal{O}_C(2)) = \deg(\mathcal{O}_C(2)) = 8$ (recall that $\pi|_E$ is a \mathbb{P}^2 -bundle and $g(C) = 1$). Hence,

$$h^0(X, \mathcal{O}_X(2L - E)) = h^0(X, \mathcal{O}_X(2L)) - h^0(E, \mathcal{O}_E(2L)) = 7$$

and $|2L - E|$ defines a morphism $\varphi : X \rightarrow \mathbb{P}^6$. Now we determine the image of X . Note that we have $L^2 \cdot E \equiv 0$, $L \cdot E^3 = \deg C = 4$, and $E^4 = \deg N_{C/\mathbb{P}^4} = 20$. Thus,

$$(2L - E)^4 = (2L)^4 - 8L \cdot E^3 + E^4 = 4.$$

Consider the exact sequence

$$0 \rightarrow \mathcal{O}_{\mathbb{P}^6}(2) \otimes I_Z \rightarrow \mathcal{O}_{\mathbb{P}^6}(2) \rightarrow \mathcal{O}_Z(2) \rightarrow 0.$$

Since $h^0(Z, \mathcal{O}_Z(2)) = h^0(X, \mathcal{O}_X(4L - 2E)) = 26$, we obtain

$$h^0(\mathbb{P}^6, \mathcal{O}_{\mathbb{P}^6}(2) \otimes I_Z) \geq h^0(\mathbb{P}^6, \mathcal{O}_{\mathbb{P}^6}(2)) - h^0(Z, \mathcal{O}_Z(2)) = 28 - 26 = 2.$$

It follows that there exist two linearly independent hyperquadrics in \mathbb{P}^6 containing Z . Since $\deg Z = (2H - E)^4 = 4$, Z is a complete intersection of two hyperquadrics.

3 Proof of Theorem 1

Denote by e a line in a fiber of the \mathbb{P}^2 -bundle $\pi|_E : E \rightarrow C$. The key to prove Theorem 1 is the following:

Lemma 2. *We have $F \cdot e = 1$.*

Proof. We denote by (e) the corresponding point in $\text{Hilb}(X)$. Let T be the reduced part of the irreducible component of $\text{Hilb}(X)$ containing (e) . Note that T is a \mathbb{P}^2 -bundle over C whose fiber T_c ($c \in C$) parametrizes lines in $E_c := \pi^{-1}(c) \simeq \mathbb{P}^2$. In particular, T is smooth and of dimension 3.

Step 1. For all $(e) \in T$ such that $e \not\subset F$, we have $\sharp(F \cap e) = 1$.² Assume the contrary, i.e. there exists $(e_0) \in T$ such that $e_0 \not\subset F$ and $\sharp(F \cap e_0) \geq 2$. Remark that $\varphi(e_0) \neq B$. Let x_i ($i = 1, 2$) be two distinct points in $F \cap e_0$ and let $b_i := \varphi(x_i)$. Consider the incidence graph:

$$\begin{array}{ccc} V & \xrightarrow{p} & X \\ q \downarrow & & \\ T & & \end{array}$$

We define $V_i := p^{-1}(E \cap \varphi^{-1}(b_i))$ and $T_i := q(V_i)$ for $i = 1, 2$. Note that $\dim V_i = \dim(E \cap \varphi^{-1}(b_i)) + 1$ because p is a \mathbb{P}^1 -bundle. We observe that $q|_{V_i}$ is a finite map. Indeed, if not, there exists $t \in T_i$ such that $q^{-1}(t) \subset V_i$. Then $e_t := p(q^{-1}(t))$ is contracted by φ . This contradicts to our assumption that E is φ -ample. It follows that $\dim T_i = \dim V_i = 2$ ($i = 1, 2$). Note also that $(e_0) \in T_1 \cap T_2$. Now, we have

$$\dim(T_1 \cap T_2) \geq \dim T_1 + \dim T_2 - \dim T = 2 + 2 - 3 = 1.$$

So, we can take an irreducible curve $A \subset T_1 \cap T_2$ passing through (e_0) . Then $q^{-1}(A)$ is a ruled surface having two exceptional curves $V_i \cap S$ ($i = 1, 2$), a contradiction.

Step 2. Consider $M := (F \cap E)_{\text{red}}$. By Step 1, we see that for each $c \in C$, $e_c := (F \cap E_c)_{\text{red}}$ is a line in $E_c \simeq \mathbb{P}^2$. So, $\pi|_M : M \rightarrow C$ is a \mathbb{P}^1 -bundle. In particular M is irreducible. We can write $E|_F = mM$ with $m \in \mathbb{Z}^+$. We have

$$(mM \cdot e_c)_F = (E|_F \cdot e_c)_F = (E \cdot e_c)_X = -1$$

By assumption, F is smooth. So, $M \subset F$ is a Cartier divisor and $(M \cdot e_c)_F$ is integer. It follows that $m = 1$, i.e. the intersection $F \cap E$ is transversal. We conclude that $F \cdot e = \sharp(F \cap e) = 1$. ■

Proof of Theorem 1. By the proof of Lemma 2, $\pi|_M : M \rightarrow C$ is a \mathbb{P}^1 -bundle and $(M \cdot e_c)_F = -1$. So, $\pi|_F : F \rightarrow F' := \pi(F)$ is the blow-up along

²We mean by $\sharp(F \cap e)$ the number of points on $F \cap e$ without multiplicity.

C , and F' is smooth. On the other hand, by [Tk], $\varphi|_F : F \rightarrow B$ is either a \mathbb{P}^2 -bundle or a Q_2 -bundle. Therefore F is a Fano 3-fold with $\rho(F) = 2$. By assumption, F is smooth. So, by the Mori-Mukai's list, the pair (F', C) is one of the following:

- (1) $F' \simeq \mathbb{P}^3$ and C is a line;
- (2) F' is a hyperquadric $Q_3 \subset \mathbb{P}^4$ and $C = H \cap H'$ with $H, H' \in |\mathcal{O}_{Q_3}(1)|$;
- (3) $F' \simeq \mathbb{P}^3$ and $C = Q \cap Q'$ with $Q, Q' \in |\mathcal{O}_{\mathbb{P}^3}(2)|$.

In the case (3), C is an elliptic curve. So, Y is a Fano manifold by [W] (Proposition 3.5). In the cases (1) and (2), we have $N_{C/F'} \simeq \mathcal{O}_C(1)^{\oplus 2}$. Since there exists an inclusion of normal bundles $N_{C/F'} \subset N_{C/Y}$, $N_{C/Y}$ cannot be isomorphic to $\mathcal{O}_{\mathbb{P}^1}(-1)^{\oplus 3}$. So, Y is a Fano manifold again by [W].

Now, by Lemma 1, we can take an extremal ray $\mathbb{R}^+[m]$ such that $F' \cdot m > 0$. Then, by Proposition 1 below, we have $\rho(Y) = 1$. In particular F' is ample. Let f be a minimal rational curve of the extremal contraction φ . We obtain the following table of intersection numbers (due to [Tk] and [MM]):

case	$F' \cdot f$	$E \cdot f$
(1)	-1 or -2	1
(2)	-1	1
(3)	-1	2

Let $f' := \pi_* f$. Note that $F' \cdot f' = (\pi^* F') \cdot f = (F + E) \cdot f$. In the cases (1) and (2), we have $F' \cdot f' \leq 0$, a contradiction because F' is ample. So, only the case (3) (in which we have $F' \cdot f' = 1$) is possible, and $(Y, F') \simeq (\mathbb{P}^4, \mathcal{O}_{\mathbb{P}^4}(1))$. Consequently, C is the complete intersection $F' \cap Q \cap Q'$ with $F' \in |\mathcal{O}_{\mathbb{P}^4}(1)|$ and $Q, Q' \in |\mathcal{O}_{\mathbb{P}^4}(2)|$. ■

It remains to prove the following:

Proposition 1. *Let Y be a smooth projective variety of dimension $n \geq 4$ and D a prime divisor on Y with $\rho(D) = 1$. Assume that there exists an extremal contraction $\mu : Y \rightarrow V$ of ray $\mathbb{R}^+[m]$ with $D \cdot m > 0$, m being a minimal rational curve of the ray. If there exists a smooth curve $C \subset D$ such that the blow-up $X := \text{Bl}_C(Y)$ is a Fano manifold, then we have $\rho(Y) = 1$. Moreover, if D is isomorphic to \mathbb{P}^{n-1} , then we have $(Y, D) \simeq (\mathbb{P}^n, \mathcal{O}_{\mathbb{P}^n}(1))$.*

Proof. We shall consider two cases:

- (1) there exists $v_0 \in V$ such that $\dim(\mu^{-1}(v_0) \cap D) \geq 1$;
- (2) $\dim(\mu^{-1}(v) \cap D) = 0$ for all $v \in V$.

In the case (1), there exists a curve $B \subset \mu^{-1}(v_0) \cap D$. So, we can write $B \equiv bm$ with $b \in \mathbb{R}^+$. Since $\rho(D) = 1$, any curve in D is numerically equivalent to a multiple of m . Hence, $\mu(D)$ is a point. We also have $D \cdot B > 0$. Now, by Proposition 4 of [T2], we conclude that $\rho(Y) = 1$.

We show that the case (2) is impossible. In this case, any fiber of μ is at most of dimension 1. So, by [A] (see also [W] Theorem 1.2), μ is either, a \mathbb{P}^1 -bundle, a conic bundle, or a blow-up along a smooth subvariety of codimension 2 in a smooth projective variety. If μ is a \mathbb{P}^1 -bundle, take a fiber m passing through a point on C . Let \tilde{m} be the strict transform by the blow-up $\pi : X \rightarrow Y$. For the exceptional divisor E , we have $E \cdot \tilde{m} \geq 1$, so that

$$K_X \cdot \tilde{m} = K_Y \cdot m + (n-2)E \cdot \tilde{m} \geq -2 + (n-2) = n-4 \geq 0,$$

which is absurd because X is a Fano manifold.

If μ is a conic bundle, the extremal rational curve m is a component of a singular fiber of μ . Let Δ be the discriminant locus and let $\tilde{\Delta} := \mu^{-1}(\Delta)$. The assumption $D \cdot m > 0$ implies $\tilde{\Delta} \cap D \neq \emptyset$. Since $\rho(D) = 1$, the non-zero effective Cartier divisor $\tilde{\Delta}|_D$ is ample. Therefore,

$$(\tilde{\Delta} \cdot C)_Y = (\tilde{\Delta}|_D \cdot C)_D > 0,$$

so that $\tilde{\Delta} \cap C \neq \emptyset$. Now, we can take a singular fiber $\mu^{-1}(v_0)$ ($v_0 \in \Delta$) meeting C . Let $m_0 \subset \mu^{-1}(v_0)$ be a component such that $m_0 \cap C \neq \emptyset$. Then, we have a contradiction as in the case of \mathbb{P}^1 -bundle. The case of a blow-up along a centre of codimension 2, can be ruled out by using a same argument for the exceptional divisor of μ in place of $\tilde{\Delta}$.

Consequently, only the case (1) is possible, so that we have $\rho(Y) = 1$. If $D \simeq \mathbb{P}^{n-1}$, by [BCW](Lemme 4) we conclude that $(Y, D) \simeq (\mathbb{P}^n, \mathcal{O}_{\mathbb{P}^n}(1))$ ■

Our assumption that $F = \text{Exc}(\varphi)$ is smooth, is used in the proof of Lemma 2 (only for *Step.2*) and in the proof of Theorem 1 in order to apply to F the Mori-Mukai's classification of smooth Fano 3-folds. So, it is natural to ask whether Theorem 1 remains true without the smoothness of F . Concerning to this question, it is worth seeing the following:

Example (A degenerate case of the example in Section 2) We consider the union of two smooth conics $C = C_1 \cup C_2 \subset Y := \mathbb{P}^4$ obtained as complete intersection of a hyperplane and two hyperquadrics. We assume that C_1 and C_2 meet at two distinct points. Let $\pi : X \rightarrow \mathbb{P}^4$ be the blow-up along the ideal $I_{C_1 \cup C_2}$ and E the exceptional divisor. Let F be the strict transform of the hyperplane containing $C = C_1 \cup C_2$. Then F is a Q_2 -bundle over \mathbb{P}^1 having exactly two ordinary double points. Remark that F is isomorphic to the blow-up of \mathbb{P}^3 along the ideal $I_{C_1 \cup C_2}$. Moreover, F can be realized as divisor in $\mathbb{P}^1 \times \mathbb{P}^3$ by the equation $sX_2X_3 + t(X_0^2 + X_1^2 + X_2^2 + X_3^2) = 0$, where $(s : t)$ (resp. $(X_0 : X_1 : X_2 : X_3)$) is the homogeneous coordinates of

\mathbb{P}^1 (resp. \mathbb{P}^3). The fiber over $(1 : 0)$ is two planes P_i ($i = 1, 2$) and the two ordinary double points lie on the line $P_1 \cap P_2$.

As in Section 2, we see that the linear system $|\pi^* \mathcal{O}_{\mathbb{P}^4}(2) - E|$ defines a $(3,1)$ -type contraction $\varphi : X \rightarrow Z$ to complete intersection of two hyperquadrics in \mathbb{P}^6 , and its exceptional divisor is F . This gives an example of (Y, C) such that $F = \text{Exc}(\varphi)$ is singular. However X is also singular along two rational curves over the two intersection points of $C_1 \cap C_2$.

4 Related results

Let X be a Fano manifold and let ι_X be its pseudo-index, i.e. the minimum of the anti-canonical degrees $(-K_X \cdot C)$ for rational curves C on X . In [BCDD], the authors discuss the inequality ("generalized Mukai conjecture"):

$$\rho(X)(\iota_X - 1) \leq \dim X$$

and prove it in dimension 4. The essential part is to show that if $\iota_X = 2$, then $\rho(X) \leq 4$. Concerning to this, we have the following:

Proposition 2. *Let $\pi : X \rightarrow Y$ be the blow-up of a smooth projective variety Y of dimension $n \geq 4$ along a smooth curve C and let E be the exceptional divisor. Assume that X is a Fano manifold and there is another blow-up $\varphi : X \rightarrow Z$ (different from π) along a smooth curve B . Let F be the exceptional divisor of φ . Then, we have $E \cap F = \emptyset$.*

Proof. Assume $E \cap F \neq \emptyset$. Take $a \in C$ and $b \in B$ such that $E_a \cap F_b \neq \emptyset$. Then we obtain $\dim(E_a \cap F_b) \geq \dim E_a + \dim F_b - \dim X = n - 4$. So, if $n \geq 5$, there is a curve contained in $E_a \cap F_b$ and then contracted by both π and φ . This is absurd because we assume $\pi \neq \varphi$. Therefore, we have $n = 4$. By (the proof of) Theorem 1, $\varphi|_F : F \rightarrow B$ cannot be a \mathbb{P}^2 -bundle. So, the case $E \cap F \neq \emptyset$ is impossible. ■

We are now able to give a simple proof of a result in [BCDD].

Theorem 2 (see [BCDD] Théorème 3.9). *Let X be a Fano manifold of dimension ≥ 4 whose birational contractions are all blow-ups along smooth curves in smooth projective varieties. Assume that X has at least one birational contraction. Then, we have $\rho(X) \leq 3$.*

Proof. Let E be an exceptional divisor on X . By Lemma 1, we can take an extremal ray $\mathbb{R}^+[f] \subset \overline{\text{NE}}(X)$ such that $E \cdot f > 0$. Then, by assumption and by Proposition 2 above, the associated contraction $\mu := \text{cont}_{\mathbb{R}^+[f]} : X \rightarrow Z$ is of fiber type. So, there is a surjection $\mu|_E : E \rightarrow Z$. Hence, we have $\rho(Z) \leq \rho(E) = 2$. Consequently, $\rho(X) = \rho(Z) + 1 \leq 3$. ■

References

- [A] T. Ando, On extremal rays of the higher-dimensional varieties. *Invent. Math.* **81**, (1985) 347–357.
- [BCDD] L. Bonavero, C. Casagrande, O. Debarre and S. Druel, Sur une conjecture de Mukai. *Comment. Math. Helv.* **78**, (2003) 601–626.
- [BCW] L. Bonavero, F. Campana and J. Wisniewski, Variétés complexes dont l'éclatée en un point est de Fano. *C. R. Math. Acad. Sci. Paris* **334**, (2002) 463–468.
- [MM] S. Mori and S. Mukai, Classification of Fano 3-folds with $B_2 \geq 2$. *Manuscripta Math.* **36**, (1981/82) 147–162.
Erratum: *Manuscripta Math.* **110**, (2003) 407.
- [S] H. Sato, Toric Fano varieties with divisorial contractions to curves. *Math. Nachr.* **261/262**, (2003) 163–170.
- [Tk] H. Takagi, Classification of extremal contractions from smooth four-folds of $(3, 1)$ -type. *Proc. Amer. Math. Soc.* **127**, (1999) 315–321.
- [T1] T. Tsukioka, Del Pezzo surface fibrations obtained by blow-up of a smooth curve in a projective manifold. *C. R. Acad. Sci. Paris* **340**, (2005) 581–586.
- [T2] T. Tsukioka, Classification of Fano manifolds containing a negative divisor isomorphic to projective space. *Geometriae Dedicata* **123**, (2006) 179–186.
- [W] J. Wiśniewski, On contractions of extremal rays of Fano manifolds. *J. Reine Angew. Math.* **417**, (1991) 141–157.

Department of Mathematics
Tokyo Institute of Technology
2-12-1 Oh-okayama, Meguro-ku,
Tokyo 152-8551, JAPAN

email: tsukiokatoru@yahoo.co.jp